Matematical correction of incomplete charge collection for photon energies up to 300 keV.

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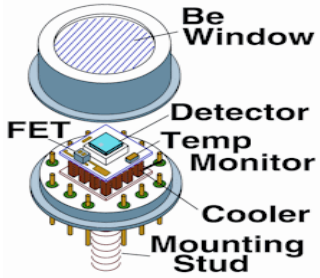
SÚRO

Semiconductor detectors based on CdTe (or CZT)

- popular in the field of gamma and X-ray spectroscopy
- high detection efficiency
- good energy resolution
- can be performed at room temperature
- disatvantage is spectral distortion, corrected by response matrix
- e.g. detection efficiency, escape effect, Compton effect

CdTe detector (AMPTEK):

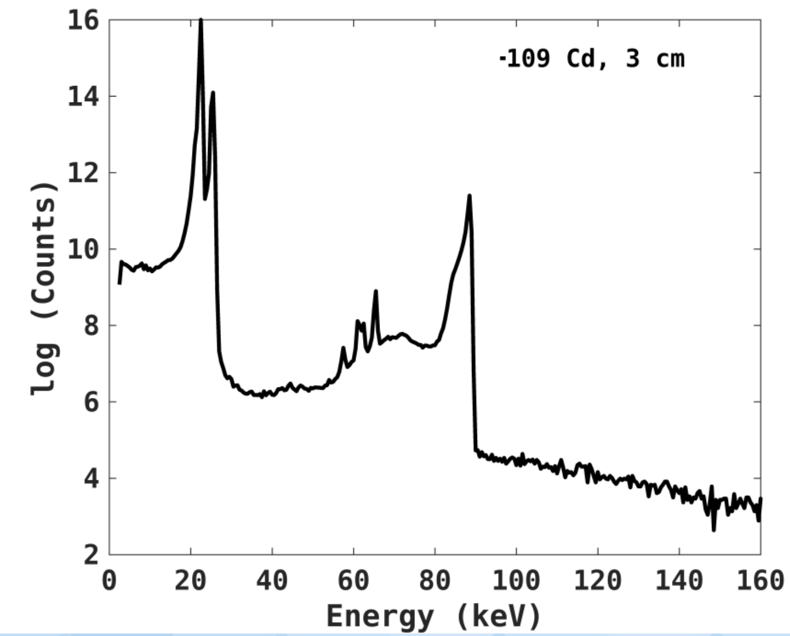


- planar crystal CdTe: 3 mm x 3 mm x 1 mm
- Peltier cooler, T: from 210 K to 215 K
- Be window: 0.1 mm
- bias voltage: 700 V
- fabricated as Schottky diode
- arrangement reduce the effect of incomplete charge collection

- multichannel analyzer, detection capability - 2.10^5 imp/s,
- detection efficiency – maximum in the range from 10 to 100 keV
- spectral distortion:
- incomplete charge collection of positive charge from crystal volume

- * * Kurková, D., Judas, L. „An analytical X-ray CdTe detector response
- matrix for incomplete charge collection correction for photon energies
- up to 300 keV.“ Radiation Physics and Chemistry 146 (2018) 26–33.

Spectrum of radioactive source ^{109}Cd (A=1.481 MBq k 20.12. 2007): 88 keV, 22.2 keV, 21.9 keV



- line 88 keV, incomplete charge collection
- = broadening towards lower energies
- $\rightarrow$ tailing

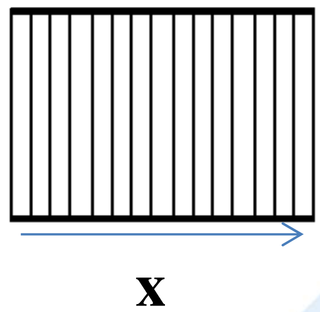
Response matrix for incomplete charge collection $T(E_j, E_k)$

- Hecht equation - model of incomplete charge collection:
- vector B = instrumental (measured) spectrum
- vector A = physical spectrum
- **B=A.T**
- **A=B.T^{-1}**
- solved as a system of linear equations without explicitely forming T^{-1}

T construction : modification of unity matrix (ideal detector)

- algorithm:
- imaginary divide crystal into n layers each assigned with F_i and E_{det}

$$F_i(E_0, \Delta x, i.\Delta x\ ) = \frac{e^{-\mu(E_0).i.\Delta x}}{\sum_{i=1}^{n}(e^{-\mu(E_0).i.\Delta x})}$$



F_i : the normalized probability of the absorption of photon of energy E_0 in the i^{th} layer

$$E_{det} = E_0 \cdot \eta(x)$$

$$\eta = \frac{\lambda_e}{D}\left(1 - e^{\left(\frac{-(D-x)}{\lambda_e}\right)}\right) + \frac{\lambda_h}{D}\left(1 - e^{\left(\frac{-x}{\lambda_h}\right)}\right)$$

Hecht equation*

x distance from cathode to the point of interaction

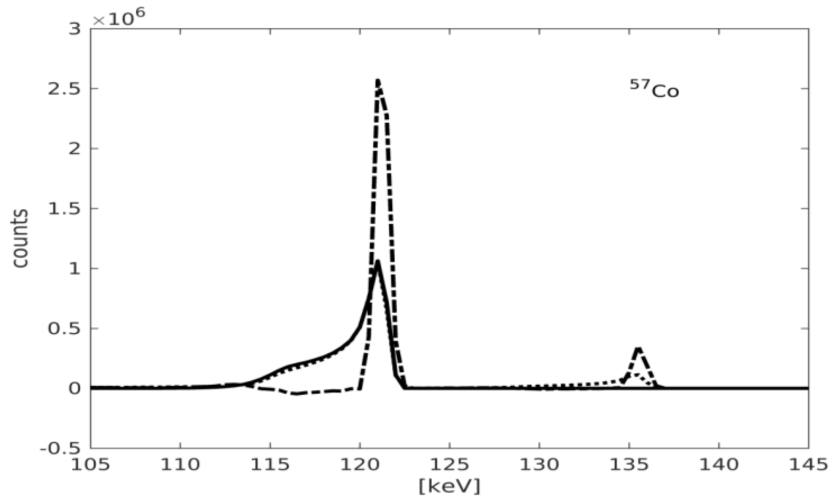
D=1 mm – crystal thickness

λ_E, λ_H mean free path of electrons and holes

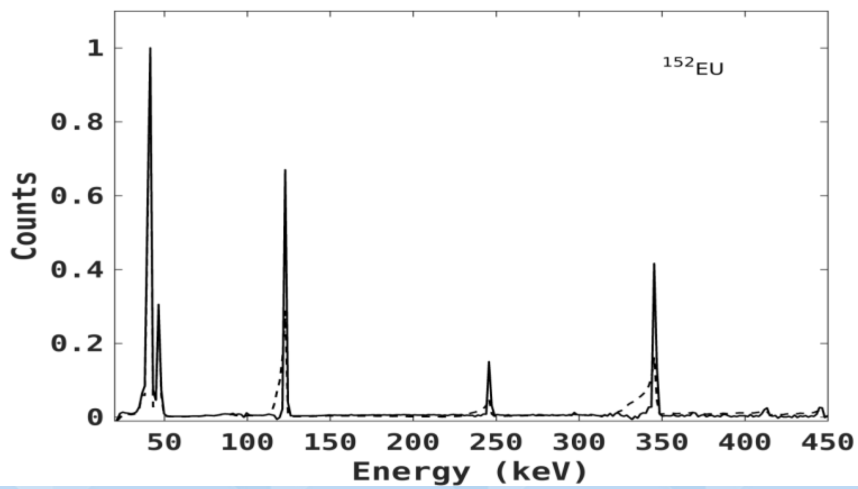
* Hecht, K., 1932. Zum Mechanismus des lichtelektrischen Primärstromes in isolierenden Kristallen. Zeit. Physik, 77, 235-245.

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

- Algoritm (continue):
- the same with unity matrix
- imaginary divide into n parts weighted with F_i
- energy shift by E_{det}

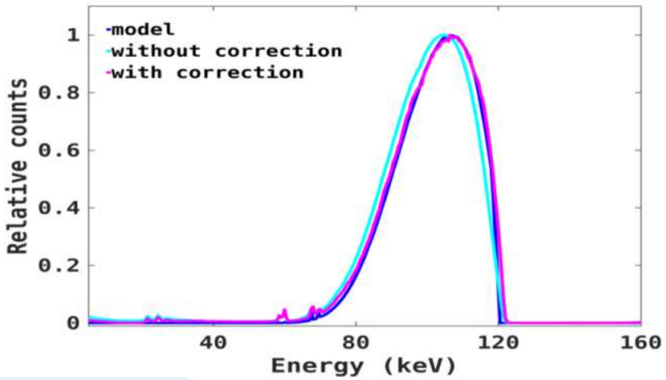


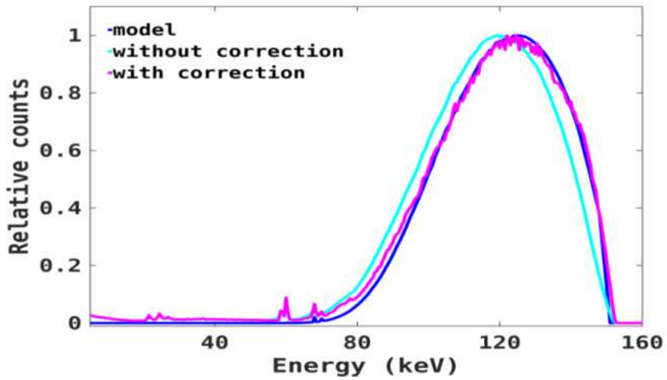
^{57}Co
solid line: model of the distortion
dotted line: measured spectrum
dashed line: inverse transformation
channel width = 0.5 keV



^{152}Eu
dotted line: measured spectrum
solid line: inverse transformation
channel width = 1.66 keV
line 344.3 keV, R: 2.4%→0.96 % (±0.28%)

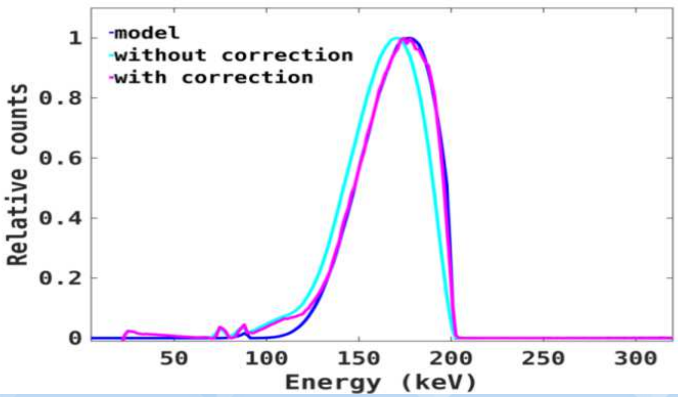
N120, N150, N200, N250, N300 (norm ISO 4037-1:1-1996)

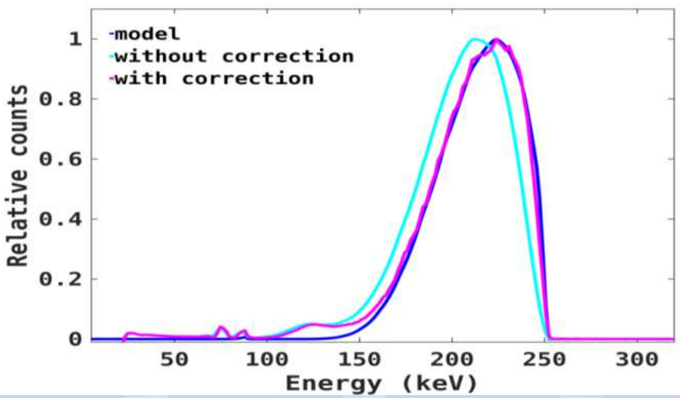


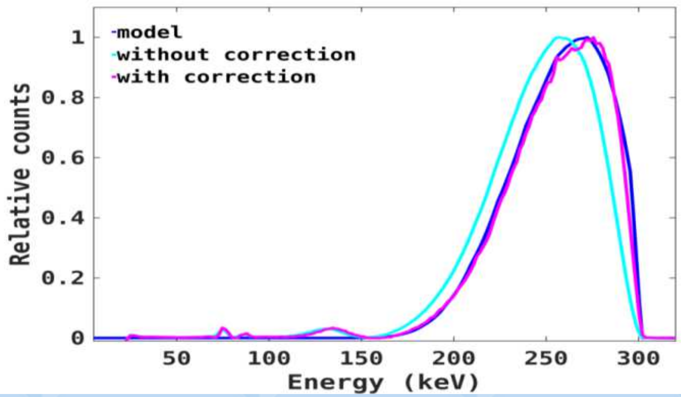


model: SpekCalc *

* Poludniowski,G., Landry, G., DeBlois, F., Evans, P.M., Verhaegen, F., 2009. SpekCalc: a program to calculate photon spectra from tungsten anode x-ray tubes. Phys. Med. Biol. 54 N433-N438







Optimization of mean free path λ_E and λ_H:

- 0 – 120 kV: λ_E = 24 cm and λ_H range from 0.4 to 1.0 cm
- 150 – 300 kV: λ_E = 24 cm and λ_H range from 0.16 to 0.65 cm
- energy dependence of this phenomenon

Spectral parameters (for quantitative analysis):

- $E_{(mean)} = \sum(E_i \cdot N_i)/\sum N_i$
- R=FWHM/*E(mean)*
- FWHM: halfwidth of bremsstrahlung
- HVL1

Emean, 1st HVL, FWHM and R, evaluated from theoretically calculated spectra and taken from the ISO norm

Beam quality	$E_{(mean)}$ (keV)		1st HVL (mm Cu)		R (%)		FWHM (keV)
	theoretical	ISO	theoretical	ISO	theoretical	ISO	theoretical
N120	101	100	1.73	1.71	28	27	28
N150	119	118	2.39	2.36	39	37	46
N200	167	164	3.90	3.99	31	30	51
N250	212	208	5.09	5.19	28	28	58
N300	254	250	5.90	6.12	27	27	70

Emean, 1stHVL, experimental and evaluated from measured X-ray spectra uncorrected and with the added correction of the incomplete charge collection.

Beam quality	experimental	uncorrected		corrected	
	1stHVL (mm Cu)	$E_{(mean)}$ (keV)	1st HVL (mm Cu)	$E_{(mean)}$ (keV)	1st HVL (mm Cu)
N120	1.76	99.0	1.65	101.0	1.72
N150	2.40	114.9	2.20	118.6	2.37
N200	4.06	160.2	3.67	165.0	3.83
N250	5.29	202.2	4.88	208.6	5.02
N300	6.24	247.8	5.81	255.8	5.97

- E(mean) v.s. ISO 4037-1:1996,
- difference from 2,8 % to 2,3% , values (< 5 %)
- E(mean) v.s. theoretical,
- difference from 4,4% to 1,4%

beam quality	ISO				theoretical			
	Uncorrected		corrected		uncorrected		corrected	
	E_{mean} (%)	1stHVL (%)	E_{mean} (%)	1stHVL (%)	E_{mean} (%)	1stHVL (%)	E_{mean} (%)	1stHVL (%)
N120	1.0	3.5	-1.0	-0.6	2.0	4.6	0	0.6
N150	2.6	6.8	-0.5	-0.4	3.4	7.9	0.3	0.8
N200	2.3	8.0	-0.6	4.0	4.1	5.9	1.3	1.8
N250	2.8	6.0	-0.3	3.3	4.4	4.1	1.4	1.4
N300	0.9	5.1	-2.3	2.5	2.6	1.5	-0.6	-1.2

Conclusion

- correction done with an analytical approach
- without the use of stochastic (Monte Carlo) method and
- without iterative or sequential mathematical methods
- model was based on the Hecht equation.
- corrected spectra are in better agreement with the theoretical model
- and the ISO

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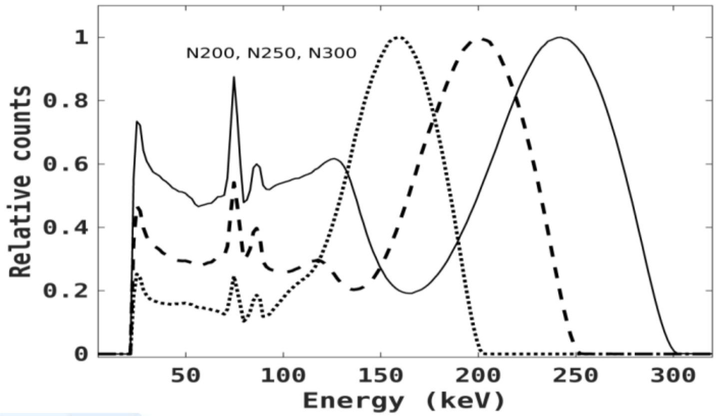

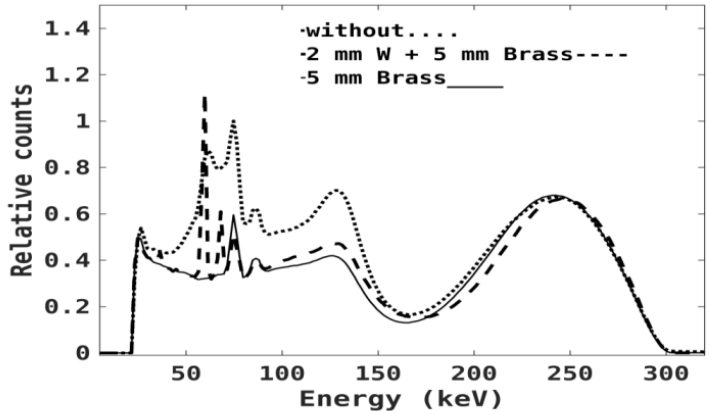

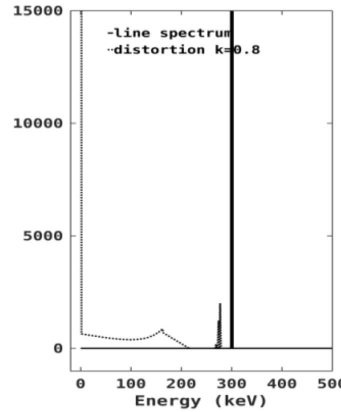

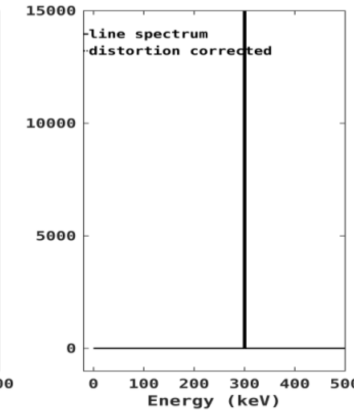

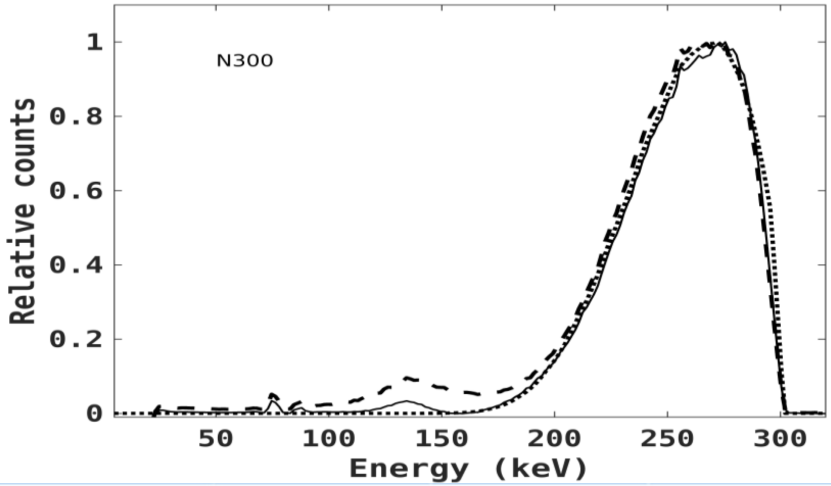

Compton scattering

linear function model of multiple scattering triangle at energies above Compton edge, for E from interval $E_c \leq E \leq 4E_c/3$

Seelentag, W.W. Panzer, W.: Stripping of X-ray Bremsstrahlung Spectra up to 300 kVp on a Desk type Computer. Phys. Med. Biol., 1979, Vol. 24., No.4, 767-780.